EM self-field theory: the electron in a hydrogen atom

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Abstract: An electromagnetic (EM) self-field theory is developed for atomic systems consisting of charged sub-particles. An azimuthal modal spinor[1] is chosen as a trial solution of the motions of each sub-particle and tested using Maxwell's equations for particle-field interactions. Both the sub-particles and field are seen in terms of coupled spinors. Unconventionally, the sub-particle electric and magnetic fields are measured between centres-of-motion due to a coupling between a sub-particle's rotational coordinate systems. Maxwell's curl equations are seen as a balance of the electric and magnetic kinetic energies with a sub-particle's total energy, and a balance of its electric and magnetic potential energies. The theory results in a system of inhomogeneous equations, the unknowns being the coupled spinors of each sub-particle, four equations for the electron in a hydrogen atom, two conjugate pairs of equations. Planck's constant is seen as a variable of motion. These four modal equations yield analytic solutions for the resonant frequency, the radii, Rhydberg's number and Balmer's series. The theory behind this solution may lead to an understanding as to how the strong and weak nuclear, the EM, and the gravitational forces, all tie in with each other

1. Introduction.

Present-day understanding of the hydrogen atom is linked to the quantum theory that evolved during several decades of effort from the late 19th century until the mid-1920's [Heisenberg 1949] [Condon and Odabasi 1980]. Quantum theory first came to light with the failure of science to provide a consistent theory for the emitted energy from a black-body radiator. The situation was resolved by Planck who realised the effect could only be treated as a series involving discrete frequencies, not a continuous function of frequency. Via the photo-electric effect, Einstein observed that radiation also acted as discrete quanta of energy $\boldsymbol{E}/\boldsymbol{h\nu}$, or photons. Bohr, recognising that the Plank-Einstein equation $\boldsymbol{E = h\nu}$ held for emitted as well as absorbed energy, put forward a quantum theory of spectroscopy in which angular momentum must be whole numbers of Planck's quantum number $\hbar = \boldsymbol{h}/2\pi$.

Applying Bohr's theory, spectral lines could be expressed as a quantum series,

$\boldsymbol{\nu_{mn} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)}$, Balmer's formula, featuring a constant, $\boldsymbol{R = \frac{q_e^4\, m_e}{8\ \varepsilon_0^2\, h^3\, c}}$,

Rydberg's number[2]. With this theory, the energy of a hydrogen atom in its ground state, the Bohr energy, $\boldsymbol{E_B = R\hbar c}$, about ***13.6 eV***, could be estimated, along with a probabilistic radial distance of the electron ***~0.529 A°,*** the Bohr radius. Spectroscopic experimentation over the next two decades lead to an understanding of the various ways in which a hydrogen atom could be excited by E- and H-fields. As at 1926, a

[1] Spacetime vectors that rotate (spin) in a single 2-D plane can be expressed in terms of a complex C_2 basis e.g. $r_o\ exp(jm\phi)$. In particle physics, spinors are two-component complex column vectors used to represent motions of particles with intrinsic half-integral spin (fermions have spin $\frac{\hbar}{2}, \frac{3\hbar}{2}, \frac{5\hbar}{2}, \ldots$).
In general, spinors are used as a basis for Hermitian matrices whose determinants are positive definite and whose eigenvalues are all real [Barut 1980]. Thus the EM field equations and the stress-energy tensor can both be expressed in terms of spinors. The term is used in this report to represent spacetime vectors that rotate extrinsically in accordance with Maxwell's equations. Thus the motions of charged particles (electrons and protons) and dipoles that perform EM rotations may both be termed spinors

[2] A list of relevant physical constants are defined and known values tabulated in the Appendix.

substantially complete quantum picture of atomic physics had emerged involving the four known quantum numbers *n, l, m*, and *s*.

Despite intensive investigation, this same period saw a failure to find any form of solution by which atomic physics could be treated by electromagnetic (EM) theory. A limiting of EM began with Maxwell's own conviction that EM could not be a basis for gravitational effects despite similarity between inverse square forms of Coulomb's electrostatic force and Newton's gravitational force [1864] [North 1960]. In atomic physics concepts of planetary motion led only to unstable spiralling of the electron. A conviction against EM as a basis for atomic physics grew as methods termed quantum mechanics (QM) evolved. Quantum electrodynamics (QED) emerged in the 1930's, a form of EM which sat under the expanding domain of QM. Planck's quanta is carried by the EM potential wave equation giving rise to a quantum wave-packet. Combining classical mechanics and electrodynamics gave a theory of commutation operators, with time-independence, the Schrödinger representation, or in general the Born-Heisenberg form [Heitler 1984]. While computational difficulties are inherent in QED exemplified by calculation of Landé's g-factor [Sethna 1997], and mass renormalization [Heitler 1984], it did provide verifiable solutions. Despite the difficulties, the standard model of the hydrogen atom resulting from the Schrödinger equation has until recently been the only option [Hofer 2000]. Against the backdrop of mathematical complexity required by QM and modern physics generally, no EM field solution based on the dynamics of charged particles has been useful in modelling a hydrogen atom apart from Bohr's early model. Studies of atomic self-fields are rare[Jackson 1962] [Barut 1980].

In this paper an EM self-field theory provides analytic solutions for the motions of the electron in a hydrogen atom including Rhydberg's number and Balmer's series. EM theory needs modification however in evaluation of the EM fields. The electric (E-) and magnetic (H-) fields are measured between centres of motion rather than point-to-point as in the Liénard-Wiechert potentials [Jackson 1962]. After testing a trial spinor form for the self-fields via Maxwell's equations, a link to quantum theory, reminiscent of QED but more physically based, is apparent. EM self-field theory gives systems of tractable equations where self-fields exist. It induces no new physics, only the mathematical apparatus needed to obtain the physics changes. Rather than QM's potentials. EM self-field theory lets an uncomplicated physical picture emerge.

2. Self-Field Theory

EM self-field theory proposes a coupled spinor solution to Maxwell's equations capable of modelling atomic systems composed of charged sub-particles. In this report the theory is specialized to an electron and a proton moving dynamically around each other to form a hydrogen atom in its various modes. The proton is assumed to move insignificantly, an 'infinite mass' proton, its motions complementing the electron's, electrically and magnetically. The spinors represent the actual motions of the electron, an orbital spinor ($\boldsymbol{v_o} = \boldsymbol{\omega_o r_o}$) associated with the E-field and a cyclotron spinor ($\boldsymbol{v_c} = \boldsymbol{\omega_c r_c}$) associated with the H-field. These spinors led to a system of equations for the electron representing a time-varying balance of its kinetic and potential energies. The equations also represent Maxwell's equations, the fields of the atomic system.

Maxwell's equations

In general, EM fields that control the motions of charged particles satisfy the Maxwell-Lorentz equations [Jackson 1962]. For application to atomic physics, the region where particle-field interactions occur can be assumed to be isotropic and homogeneous; the constitutive parameters, ε_0 and μ_0 the permittivity and permeability of free-space, are invariant scalars. Where spatially isolated sub-particles carrying units of elementary charge of opposite polarity are to be studied, in the absence of nebular regions of charge and current density, Maxwell-Lorentz equations can be written

$$\nabla \times \vec{E} + \mu_0 \frac{\partial \vec{H}}{dt} = 0 \tag{1a}$$

$$\nabla \bullet \vec{H} = 0 \tag{1b}$$

$$\nabla \times \vec{H} - \varepsilon_0 \frac{\partial \vec{E}}{dt} = \frac{\pi}{s_q} q \vec{v} \tag{1c}$$

$$\nabla \bullet \vec{E} = \frac{4\pi}{v_q} q \tag{1d}$$

$$\vec{F} = q\vec{E} - q\vec{v} \times \vec{B} \tag{1e}$$

along with the constitutive equations $\vec{B} = \mu_0 \vec{H}$, $\vec{D} = \varepsilon_0 \vec{E}$. Eqns 1a-d are termed the classical EM field equations[3]. It is assumed that the volume of integration (v_q) over which the charge density is evaluated, and the area the charge circulates normal to its motion (s_q), are constant during sucessive periods over which the internal motions of the atom take place. The period is not assumed circular or in any way regular apart from being periodic. These parameters need to be retrospectively examined to check the validity of solutions that are uncovered for a particle's motion. Non-singularity of charge- and mass-points follow from the assumption that a sub-particle never resides at the origin due to the form of spinors governing its motions. Forces acting upon a sub-particle are given by the specialized Lorentz Eqn 1e where the current loops of the two interacting particles are considered to move in the same direction of phase.

Maxwell's equations can be used to specify the spatial distribution and motions of photons that comprise the E- and H-fields due to the presence of charged particles. The concept of the field and the charged sub-particles within it is that of a collection of smaller particles (photons) that transit between larger particles (electrons and protons). We may think of the visual capability of computers to 'zoom' into a region being displayed. At afar, the field looks like a nebulus cloud. As we zoom in,

[3] Where a current density is used in Eqn 1c, the factor 4π in the Biot-Savart law comes about from an application of Green's theorem leading to a surface over the volume enclosed by the charge density. For the case of discrete charges, the factor π represents the area enclosed by the moving charge-point.

the cloud comprising the field becomes a series of discrete point-like particles, the photons. At the same time the sub-particles, the electron and the proton, may change their visual character from point-charges to objects with internal dynamic structure. The E- and H-fields acting on the electron are a measure of a photon exchange process between proton and electron. The spinor motions are related to this exchange. The orbital and cyclotron motions may be visualized as a series of tiny photon/electron interactions which change the electron's direction (and vice versa). The overall physics is complex but over each cycle obeys the differential geometry embodied by Maxwell's equations.

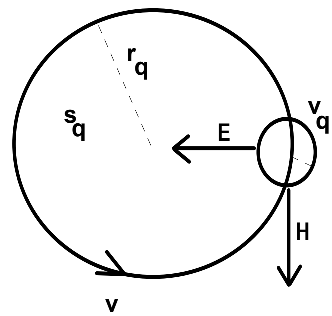

Fig. 1 Electromagnetic forces acting upon a charged atomic sub-particle where Maxwell's equations are specialized for oppositely charged interactions as between proton and electron.

A trial solution for the self-fields

The process of solving differential equations may sometimes be shortcircuited by what may be called inspired guesswork . Where a form is suspected, solutions given by the right hand side of the equations are compared with those given by the left hand side. Such an intuitive approach is used to solve Maxwell's equations for the self-fields. The trial solution is based on mathematical forms associated with the effects of isolated E- and H- fields upon charged particles. Charged particles perform cyclotron motion, a spinor solution to Maxwell's equations, when acted upon by a fixed uniform H-field. Similarly a fixed central E-field can also act on a moving charged particle to yield a spinor solution (termed orbital motion), albeit in this case an unstable solution. We speculate if a system comprising both an E-field and an H-field i.e. two particles in relative motion wrt each other, can be considered as a coupled pair of spinors. E- and H-fields are complimentary, and it is no surprise that a coupled spinor can be found that exactly satisfies the differential geometric relationship of Maxwell's equations. This coupled spinor solution will be applied to the electron in a hydrogen atom.

A standard mathematical procedure in EM applications involves separating the EM variables into azimuthal modes. Where axisymmetry of the medium and field holds, this can be done using single mode analyses [Harrington 1961] [Jackson 1962]. In more general cases coupled azimuthal modes can be used [Mei 1974][Fleming 1990]. Initially in the following, only the main principle mode, $\boldsymbol{m = 1}$ $(\boldsymbol{e}^{j\,\phi})$ is investigated. A complete azimuthal modal $(\boldsymbol{e}^{j\,m\phi})$ analysis follows readily from the analytic solution for the main principle mode case. Following Von Hippel [1962, p.45] a functionally separated form for the electron orbital motion is assumed as a radially directed spinor rotating in the azimuthal angle ϕ as shown in Fig.2:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} e^{j\phi} \tag{2}$$

The scalar potential φ is relate d to this spinor.

$$\varphi = \frac{-1}{4\pi\varepsilon_0} \frac{q}{r} e^{j\phi} \tag{3}$$

The time-derivative of the H field can be formed from the spherical curl of the E-field. Assuming a rotational dependence $\phi = j\omega t$, integration wrt time reveals an H-field spinor form that also rotates in ϕ ($H = \frac{1}{4\pi\varepsilon_0\mu_0} \frac{q}{\sin\theta \ \omega r^3} e^{j\phi}$). This can also be obtained from the vector and scalar potentials, $\frac{\partial A}{\partial t} = \frac{-1}{\mu_0}(E + \nabla\varphi)$. To here the solution is consistent. The E- and H-fields that transverse space can be represented as spinors related by a similar radial dependence. The cycle of differentiation implicit in Maxwell's equations is completed to see if the form of the resulting E-field is consistent with the original trial form (Eqn 2). Taking the curl of the time-derivative of the H-field where $c^2 = 1/\mu_0 \varepsilon_0$ we obtain

$$\nabla\times\frac{\partial H}{\partial t} = \frac{-1}{4\pi} \frac{c^2 q}{\sin\theta^2 r^4} e^{j\phi} \hat{r} + \frac{c^2}{2\pi} \frac{jq}{\sin\theta^2 r^4} e^{j\phi} \hat{\phi} \tag{4}$$

Differentiating Eqn 2 twice wrt time where a rotational dependence $\phi = j\omega t$ is again assumed reveals its similarity in form to the first term on the rhs of Eqn 5:

$$\frac{\partial^2 E}{\partial t^2} = \frac{-1}{4\pi \ \varepsilon_0} \frac{\omega^2 q}{r^2} e^{j\phi} \hat{r} \tag{5}$$

Eqns 4-5 are components of the total current density (Eqn 1c) differentiated wrt time. Summarising, a radial E-field spinor rotates in ϕ; the E-field and all its time derivatives form a spinor set in the $r-\phi$ plane. A θ directed H-field derivative spinor also rotates in ϕ; the H-field and all its derivatives form a set in the $\theta-\phi$ plane. The H-field spinor sets up cyclotron forces upon the proton in turn causing the electron to perform cyclotron motion in the $r-\theta$ plane (see Figs 2-4). Given certain conditions which we need to investigate, this coupled spinor is a self-field solution to Maxwell's equations. Remembering that we interested at present in the main principle mode, note that the two motions at all times move relative to their respective centres of motion and in a synchronous fashion relative to each other.

These motions while interdependent according to Maxwell's equations are mathematically orthogonal.

Examining Eqns 4 and 5, a condition for the coupled spinor solutions to cycle theoretically is $v^2 / \sin^2\theta = c^2$ where $v = \omega r$ with an obvious precondition $\sin\theta = 1$ or $\theta = \pi/2$ rad. The rotations thus occur at the equatorial elevation, leaving the condition $v = c$. To this point the EM equations have been used to describe the fields regardless of their application and involve the speed of light.

The spinors $v_o = \omega_o r_o$ and $v_c = \omega_c r_c$ are thus introduced as variables into Maxwell's curl equation where ω_o and ω_c are the orbital and cyclotron angular velocities, r_o the orbital radius due to the E-field and r_c the cyclotron radius due to the H-field. These orbital and cyclotron spinors represent solutions to Maxwell's divergence equations. A system of equations based on the EM equations can be written where the orbital area constant $s_o = \pi r_o^2$ and the cyclotron area constant $s_c = \pi r_c^2$ and where we shall assume that $|\omega| = |\omega_o| = |\omega_c|$:

$$\frac{c^2}{4\pi}\frac{jq}{r_c^3} = \frac{-c^2}{4\pi}\frac{jq}{r_o^3} \tag{6}$$

$$\frac{1}{4\pi}\frac{c^2 q}{\omega r_c^4} + \frac{1}{4\pi}\frac{\omega\, q}{r_o^2} = \frac{1}{2s_o}\, q\omega \tag{7}$$

$$\frac{c^2}{2\pi}\frac{q}{\omega r_c^4} = \frac{1}{2s_c} q\omega \tag{8}$$

The coupled spinor solution, Eqn 6-8, follow from Maxwell's curl equations, Eqns 1a and 1c, with $c^2 = 1/\mu_0\,\varepsilon_0$ being the only simplifaction. As stated previously, the E- and H-field spinors result from Maxwell's divergence equations, Eqns 1b and 1d. Overall there is a dynamic balance of the E- and H-forces, Eqn 1e. It must be remembered that the E- and H-fields are being measured between centres of motion and not point-to-point. These equations, 6-8, indicate certain systems of charged sub-particles are self-sustaining if they involve a rotation speed equal to the speed of light; an important side issue [Heitler 1954 p. 324] which is left aside in this report. Here the aim is to investigate whether the spinors might form a possible solution to the internal dynamics for the electron of a hydrogen atom. As such, the rotational velocities of the electron involve speeds of the order 1/137th that of light. Implicit in our solution has been the scaling factor $c^2 = 1/\mu_0\,\varepsilon_0$. Perhaps a similar solution might be able to describe atomic systems where the electron's velocity may be written using a fine-structure parameter $v_e^2 = \alpha_e^2/\mu_0\,\varepsilon_0$. Assuming that an atomic sub-particle's motions form a solution to the EM equations, the internal dynamics for the electron may again be written in terms of coupled spinors:

$$r(r_o, \theta_o, \phi_o, r_c, \theta_c, \phi_c, t) = r_o \exp(j\omega_o t) + r_c \exp(j\omega_c t) \qquad (9)$$

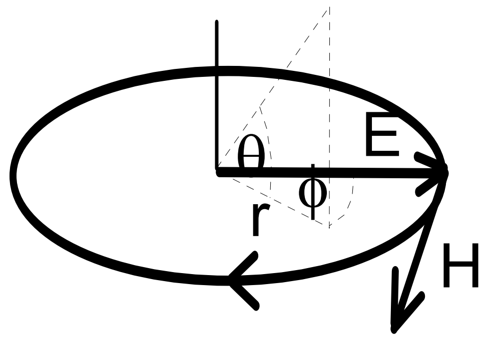

Fig. 2 Coupled spinor solution: E- and H-fields rotating in ϕ plane.

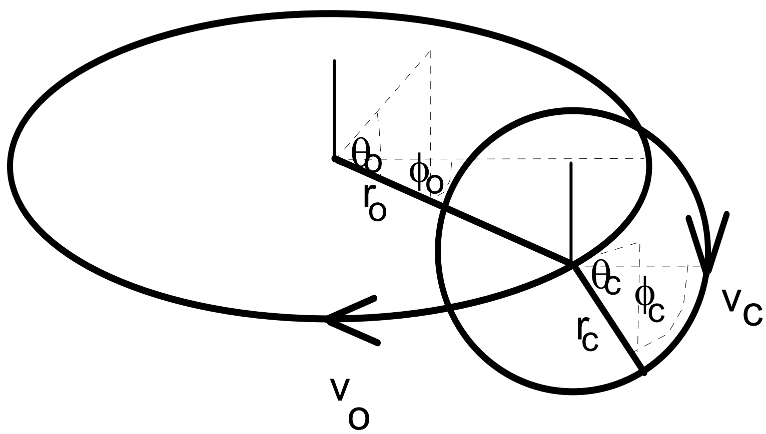

Fig. 3 Orbital and cyclotron radii and velocities of electron showing spherical coordinate system oriented at the electron's electrical centre of motion (centre of mass). The coordinate systems of the orbital and cyclotron motions can be differentiated from each other. Only four of the six spactial coordinates are independent, as in space-time coordinate systems.

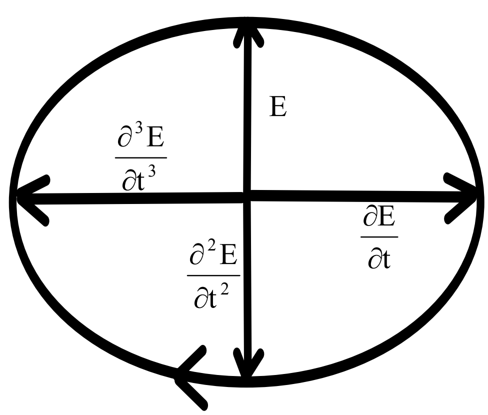

Fig. 4 The time derivatives of the EM spinor fields form series, n=0,1,2,3, that return to the initial direction after a complete cycle. Shown is the E-field series which lies in the r-ϕ plane

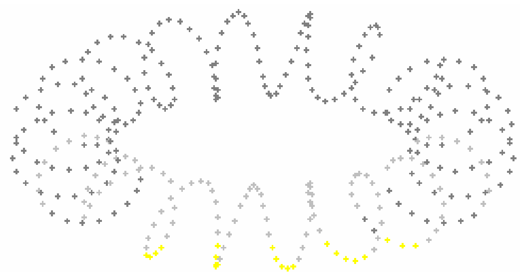

Fig. 5 Coupled spinor solution showing the motions of the E- and H-field spinors rotating in ϕ plane where $r_o = 4r_c$ and $\omega_c = 20\omega_o$. Orbital and cyclotron motions are a combined motion; each spherical component has its own centre of motion (origin), one stationary, the other moving.

What needs to be realised with the spinor form of Eqn 9 is that the resulting position of the particle is indeed the sum of two spinors. Each spinor refers to one of two centres of motion. While the centre of motion of the E-field spinor is stationary, that of the H-field rotates along with the E-field spinor. This results from the coupled nature of the two spherical coordinate systems shown in Fig. 3. Figs 2-4 illustrate the mechanics of the spinors, but they do not show the actual positions mapped out by them. Shown in Fig.5 shows one possible motion where $r_o = 4r_c$ and $\omega_c = 20\omega_o$.

One important aspect of the spinor solutions is their rotational basis in time. In terms of mathematical solution forms which lead to discrete or decoupled azimuthal modes, the complex exponential form imbedded in Eqn 9 is one such form. Those solutions which return to their starting point, in other words are periodic, may be capable of maintaining a stable dynamic motion capable of continuing a self-motion without net efflux or influx of energy. Thus azimuthal modal forms which can be written as $exp(jm\phi)$ where $j = 1,2,3,...$ are a prerequisite to a discrete physics.

3. A system of self-field equations for a hydrogen atom.

Circular and elliptic motions can establish a balance between centripetal and centrifugal forces as is well known from the study of electrostatic and gravitational orbits. Where an electron and a proton are in dynamic equilibrium, pairs of electrical and magnetic forces each balance the mass of the sub-particle in orthogonal directions of motion[4].

'Infinite mass' proton

Assuming the E-field spinor form (Eqn 2), and an 'infinite-mass' proton, an equation can be written for the electron's orbital motion:

$$m_e \frac{d\ r_o^2}{d^2 t} = q\ \ E \tag{10}$$

$$m_e \frac{d\ r_o^2}{d^2 t} = \frac{1}{4\pi\varepsilon_0}\ \frac{q^2}{r_o^2} e^{j\phi} \tag{11}$$

[4] It appears entirely possible that a dipolar spinor model for the neutron can accomodate a similar balance of forces and energies as exists between electron and proton.

where m_e is the mass of the electron. The charges being equal and opposite, the resulting E-field is attractive and the orbital centripital force is given by $\boldsymbol{m_e \omega^2 r_o}$. How do the motions of the electron and proton establish a constant E-field spinor? Leaving aside for the moment the 'infinite mass proton' assumption, the two rotate around a common stationary centre of motion. A constant orbital separation can be maintained provided (a) the two maintain a certain prescribed angular velocity in ϕ, and (b) their ϕ rotations are in antiphase with each other (their phases differ by π). On its own such an orbital system would be unstable as is known [Von Hippel 1962].

Using the derived H-field spinor form, Eqn 4 (integrating wrt time), and again assuming an 'infinite-mass' proton, a second equation identical in form to Eqn 12 can be written for the electron's cyclotron motion:

$$\boldsymbol{m_P \frac{d\ r_c^2}{d^2 t} = q\ v_c \times B} \tag{12}$$

$$\boldsymbol{m_P \frac{d\ r_c^2}{d^2 t} = \frac{-1}{4\pi\varepsilon_0}\ \frac{q^2}{r_c^2} e^{j\phi}} \tag{13}$$

where the charges due to the electron and proton rotate in the same direction and hence the cyclotron force $\boldsymbol{m_e \omega^2 r_c}$ is repulsive (centrifugal). Again, a constant cyclotron radial separation can be maintained provided: (a) both particles maintain a certain angular velocity in θ, and (b) their θ rotations differ by π. On its own such a cyclotron system would be impossible as well as unstable as it requires the E-field for its existence which in turn requires the H-field; together they form the coupled EM field. When thus considered the concept of photons transitting between sub-particles appears reasonably natural using a simple addition of spinor velocities, and no complex field formulations are needed as in the case of the point-to-point Liénard-Wiechert potentials [Jackson 1962]

A system of inhomogeneous equations for the dynamic motion of the electron in a hydrogen atom can be written in terms of the spinors $\boldsymbol{v_o = \omega r_o}$ and $\boldsymbol{v_c = \omega r_c}$ via Maxwell's two curl equations in which the current densities associated with the orbital and cyclotron motions are separated into their two scalar components. In the principle mode series both spinors have equal magnitude, $|\omega| = |\omega_o| = |\omega_c|$:

$$\boldsymbol{\frac{1}{4\pi}\frac{\omega_o^2 q}{r_o} = \frac{-1}{4\pi}\frac{\omega_c^2 q}{r_c}} \tag{14}$$

$$\boldsymbol{\frac{1}{4\pi}\ \frac{\omega_o q}{r_o^2} + \frac{1}{4\pi}\frac{v_o v_c q}{\omega_c r_c^4} = \frac{1}{2 s_o}\ q\omega_o} \tag{15}$$

$$\frac{1}{2\pi}\;\frac{v_o v_c q}{\omega_c r_c^4} = \frac{1}{2s_c} q\omega_c \qquad (16)$$

where the electron's speed is assumed related to the speed of light and the constitutuve parameters as in Eqns 6-8 $\alpha_e^{\,2}c^2 = \alpha_e^{\,2}/\mu_0\,\varepsilon_0 = v_{o,e}v_{c,e}$. Two other choices are also possible $\alpha_e^{\,2}c^2 = \alpha_e^{\,2}/\mu_0\,\varepsilon_0 = v_{o,e}^2$ or $\alpha_e^{\,2}c^2 = \alpha_e^{\,2}/\mu_0\,\varepsilon_0 = v_{c,e}^2$ but the coupled form is in keeping with the coupled differentiation of Maxwell's equations. Eqn 14 is satisfied if as well as having equal magnitude, there is a phase difference of $\frac{\pi}{2}$ in real phase space, or equivalently, $\pm j$ in Fourier space, between the angular frequencies ω_o and ω_c. This relationship is effectively a condition on the phases at which both the photons and the sub-particles must interact in order to conjointly satisfy Maxwell's equations.

The virial theorem [Vasil'ev and Lyuboshits 1994] links kinetic and potential energy ($T = - < V > / 2$).This lets us rewrite Eqns 14-16 for the principle mode. Eqn 1a is thus seen as a balance between electric and magnetic potential energies and Eqn 1c a balance between electric and magnetic kinetic energies and the total energy:

$$\left|\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r_o}\right| = \left|\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r_c}\right| = 2\hbar\nu \qquad (17)$$

$$\tfrac{1}{2}m_e v_o^2 + \tfrac{1}{2}m_e v_c^2 = 2\hbar\nu \qquad (18)$$

$$m_e v_c^2 = 2\hbar\nu \qquad (19)$$

where discrete quanta of Planck's energy form the rhs of Eqns 17-19. This connection to quantum theory is obtained directly from Eqns 15-16 where $\hbar = \frac{q^2}{8\pi\varepsilon_0 v_0} = \frac{q^2}{8\pi\varepsilon_0 v_c}$ for the case $\omega_o = \omega_c$. In this form, Planck's 'constant' is a variable of motion, dependent on the solution of the equations. The quantum nature of the electron's motion has long been observed [Thompson 1928]. We have a system of two equations in the unknowns, ω and r_o, or four equations in the unknowns ω_o, ω_c, r_o and r_c. Eqns 17-19 are based on classical energy forms. These relationships may be written in matrix form including the virial theorem in the four energy components

$$\begin{bmatrix} \omega_o^2 & -\omega_c^2 & & \\ 1 & 1 & -2 & -2 \\ & & 1 & 1 \\ & & & 2 \end{bmatrix} \bullet \begin{bmatrix} V_o \\ V_c \\ T_o \\ T_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2\mathrm{h}\nu \\ 2\mathrm{h}\nu \end{bmatrix} \qquad (20)$$

where V_o , V_c , T_o , and T_c are the orbital and cyclotron component of potential and kinetic energies. This form is suitable for situations where the angular frequenices associated with the modes are not equal $\omega_o \neq \omega_c$.

Principle mode solution – Rhydberg's Number

Using Eqns 17-19, two equivalent systems of equations result one for the orbital motion, the other for the cyclotron motion. These two systems are readily solved analytically and allow direct compared with expresssions obtained from Bohr theory for the Bohr radius, the frequency at resonance and Rhydberg's number:

$$r_o = r_c = \frac{4\varepsilon_0 \hbar^2}{\pi m_e q^2} = 4r_B \qquad (21\}$$

$$\omega_o = \omega_c = \frac{\pi q^4 m_e}{16\varepsilon_0^2 \hbar^3} \qquad (22)$$

$$R_o = R_c = \frac{q^4 m_e}{32\varepsilon_0^2 \hbar^3 c} = R/4 \qquad (23)$$

From Eqn 21, the orbital and cyclotron radii are $4r_B$. From Eqn 22, the orbital and cyclotron angular frequencies are $\omega_e = 1.033532 \times 10^{16}\ rad\ sec^{-1}$.

If the two components of energy from the electron's motion are combined with two equal components of energy from the proton's motion, we obtain the frequency $f_e = 6.579671 \times 10^{15}\ sec^{-1}$, or a wavenumber 10,973,710. In this way the total system energy is linked to the motions of both the electron and proton. So as the electron changes state, the motion of the proton complements any such change.

Complete azimuthal mode solution – Balmer Series

Repeating the principle mode analysis using higher order discrete azimuthal modal series for both the H-field ($e^{j\,m\phi}$) and the E-field ($e^{j\,m\phi}$)spinors readily provides the Balmer series. This can be seen directly from Eqns 4-5:

$$\nabla \times \frac{\partial H}{\partial t} = \frac{-1}{4\pi} \frac{m^2 \omega^2 q}{r^2} e^{j\,m\phi}\, \hat{r} + \frac{m^2 \omega^2}{2\pi} \frac{jq}{r^2} e^{j\,m\phi}\, \hat{\phi} \qquad (24)$$

$$\frac{\partial^2 E}{\partial t^2} = \frac{-1}{4\pi\ \varepsilon_0} \frac{m^2 \omega^2 q}{r^2} e^{j\,n\phi}\, \hat{r} \qquad (25)$$

This immediately gives the well-known analytic form of the spectroscopically obtained Balmer series which represents the energy emitted when an atom shifts between two azimuthal modes. This series is also well known to quantum theory.

$$\nu_{mn} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \tag{26}$$

Discussion

The theory has assumed an 'infinite-mass' proton. It is possible to use a 'finite-mass' proton with the same theory and arrive at a system of equations which provides solutions for the motions of the proton modelled as another pair of coupled spinors in similar fashion as has been applied to the electron. This is left for a separate report.

An important question concerns the stability of the EM spinor solution given the well-known instability of electrostatic theory [Von Hippel 1962]. The total energy which exhibits a non-trivial stability point in phase space may be written as:

$$E_{Tot} = \tfrac{1}{2} m_e\, v_o^2 + \tfrac{3}{2} m_e\, v_c^2 - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r_o} + \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r_c}\frac{r_o}{r_c} \tag{27}$$

Another question arises as to where the self-energy of the system comes from. The theory indicates that the photons supply the energy to the electron and the proton and vice-versa. The fact that discrete quanta of Planck's energy are involved in the particle-field interaction strongly implicates the photon as the energy provider for the atomic system. The dynamics for the atomic system can be written as Eqns 17-19, an inhomogenous system where the rhs is the system energy. In addition to representing Planck's quanta, these are derived from the current densities of Maxwell's equations (Eqn 1c). This suggests two interfacing systems of motion each supplying the energy for the other; two interfacing inhomogeneous systems of equations.

As stated previously, the E- and H-fields are calculated unconventionally in terms of two coupled coordinate systems. Thus electric and magnetic forces act through differing points in space. In conventional EM theory fields are calculated between charge points. Jackson considers current loops and the Biot-Savart law in terms of Newton's third law relating to the requirement that if a force acts between two objects then the force acts through a straight line joining the two objects, a line joining both charge-points [1962]. Newton's laws can be paraphrased as follows:

LAW I. *A particle remains at rest or continues to move in a straight line with a uniform velocity if there is no unbalanced force acting on it.*

LAW II. *The acceleration of a particle is proportional to the resulting force acting on it and is in the direction of this force.*

LAW III. *The forces of action and reaction between contacting bodies are equal in magnitude, opposite in direction, and are collinear.*

Why then are the forces no longer point-to-point? Perhaps this is because: (a) the electron and the proton are not in physical contact with each other as required by LAW III which was designed for the study of statics, rather photons act as the field intermediary of the forces between the two, and (b) interactions between photons and the electron or the proton occur in a statistical fashion across the full range of a sub-atomic particle's motions. When these tiny interactions are averaged out they represent the E-and H-fields which can be represented by forces acting upon centres of motion. Thus we can suggest a fourth law concerned with dynamic equilibrium:

LAW IV. *Where non-contacting bodies are in dynamic equilibrium with each other, the electric and magnetic forces are equal in magnitude, opposite in direction, and are collinear between the origins of their orbital and cyclotron frames of reference.*

The modal analysis performed in this report has not been fully comprehensive There are other possible modes in addition to the principle mode whereby a 'fine-structure' is revealed. By repeating the analysis using a double modal series, one for the E-field ($e^{j\,m\phi}$), and one for the H-field ($e^{j\,n\phi}$), the separation into a second mode is apparent where the energy states occur much closer together than in the principle or azimuthal mode. The energy levels are separated by a quantum factor $\alpha h\nu$ involving the fine-structure constant α . In fact using either form of the energy Eqns 17-19, or Eqn 20, four separate modes are seen where each mode of energy, the two potentials, and the two forms of kinetic energy, can give rise to separate states of energy. This modal energy structure having four degrees of freedon may well correspond to experimental data provided by spectroscopy and theory provided by quantum theory.

It was suggested earlier in this report that the neutron might be modelled using a dipolar form of the spinors that successfully model a hydrogen atom. A spinor form for the E-field ($= \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}e^{j\,\phi}$) led to spinor forms to represent the motions of each sub-particle, an orbital spinor ($v_o = \omega_o r_o e^{j\phi}$) associated with the E-field and a cyclotron spinor ($v_c = \omega_c r_c e^{j\phi}$) associated with the H-field. A dipolar form for the spinor E-field, $E_{dip} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{dq}{r^3}e^{j\phi}$ where d represents the dipole moment of an electrically neutral atomic sub-particle such as a neutron, will lead to a similar analysis as presented in this report. Thus a virial mass may be related to the angular frequency of a dipolar sub-particle of mass, m, rotating along with electrons and protons. As in gravitational systems, where many atomic particles can be considered dipolar, a similar dynamics based on a dipolar form of the self-field equations may exist. Perhaps the first question with such a theory based in electric and magnetic dipoles is how such a system might be attractive since dipoles can be either attractive or repulsive. It is not known whether gravity is indeed repulsive or attractive but often it is reasonably stable. Not until an analysis can demonstrate whether there exist stable configurations of dipole spinors can we know which configuration might enable the lowest energy state.

Detailed comparison of EM self-field theory and quantum theory is best left to a separate report. Briefly, there is an intimate relationship between these theoretical models, both capable of modelling a hydrogen atom. The spinor equations, Eqns 14-

16, and energy equations, Eqns 17-20, in particular the relationship between their rhs and Planck's energy quanta are reminiscent of the theoretical basis of QED. The difference is QED's reliance on a mixed formulation involving two conjugate potentials from which observables such as energy shifts between allowed stated. Compared to this, EM self-field theory provides a separation of motion and energy into uncoupled mathematical forms. Therefore, unlike QM that cannot provide deterministic motions but must provide probabilistic solutions, there is no uncertainty in EM self-field theory.

The EM self-field equations are capable of providing a spinor solution to Maxwell's equations for the electron (and proton) in a hydrogen atom. At the same time it was noted that the trial solution also applies to the photon. The possibility exists that the photon has a similar self-field structure to the atom. They also may have two sub-particles that revolve around each other providing a balance of their dynamic energies. Further, since the energy response of the photon is continuous, the two sub-particles are required to have equal mass. While the mass of the photon is not known beyond its lower limit[5] [Jackson 1998], it can be seen that the EM-field itself may obey similar rules to atomic physics in that there are regions of enhanced energy where photonic compounds may exist. There may exist relatively large amounts of energy in the EM self-field that 'integrates' as the nucleus is approached. This may be due to a higher energy density of the region near the slower moving nucleus relative to that near the electrons which move much faster sweeping out higher surfaces of rotation. Such concepts may provide resolution as to how the strong and weak nuclear forces, the electrons, and the self-fields, all contribute to the overall energy of atomic structures ($\boldsymbol{E} = \boldsymbol{mc}^2$).

Conclusion

In this report it has been demonstrated that despite a century of suggestions to the contrary, it is possible to successfully use EM theory as a basis for the physics of a hydrogen atom. An EM self-field theory was demonstrated to be theoretically cyclic. Applying this self-field theory to a hydrogen atom, the dynamics of the electron was modelled as two coupled spinors governing its orbital and cyclotron motions. The resulting EM self-field equations and their solutions were shown to be consistent with spectroscopic data and quantum theory. Analytic expressions were obtained for the frequencies and radii at resonance, Rhydberg's number and the Balmer series; no new physics is suggested by the formulation, only a simpler, clearer version of existing data.

Apart from the obvious spectroscopic applications, any knowledge that EM self-field theory provides, above that given by QM, may be useful in applications to energy and noise. Cleaner production of energy may be possible using EM self-field theory as a basis for a dynamical chemistry, provided the neutron can be modelled with the dipolar spinor forms outlined in the discussion. Physical measurements may

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[5] At present, the best estimate of the lower limit of the photon's rest-mass is $m_{ph} < 4 \times 10^{-51} kg$.

be less noisily determined by knowing precisely where the atomic sub-particles are located.

As suspected by many, a unification of the basis for the four known physical forces, gravitation, electromagnetics, weak and strong nuclear, may be possible using field theory. That such a unification may turn out to be EM based depends on many subsequent investigations using the EM self-field theory outlined in this report. Such a unified field theory based on EM self-field theory would link processes from a very wide range of physical processes involving the four known forces and including the five known states of matter, solid, liquid, gas, plasma and Bose-Einstein condensate. Part of this unification might involve a review of the physical basis of special and general relativity in the light of the theory of the spinors presented in this report.

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Appendix:

Physical constant	**Symbol**	**Known value [Krane 1988]**
Speed of light	c	2.99792458×10^{8} m s^{-1}
Fundamental unit of charge	q	1.602189×10^{-19} C
Permittivity of free space	ε_0	$10^7/(4\pi c^2)$
Rest mass of electron	m_e	5.485803×10^{-4} x u
Mass scale factor relative to C_{12}	u	1.660566×10^{-27} kg
Bohr radius	r_0	$0.5291771 \times 10^{-10}$ m

Table 1: Table of Physical Constants.